ADAPTIVE MODULATION FOR MULTI-ANTENNA TRANSMISSIONS WITH PARTIAL CHANNEL KNOWLEDGE

[0001] This application claims priority from U.S. Provisional Application Serial No. 60/499,754, filed September 3, 2003, the entire content of which is incorporated herein by reference.

TECHNICAL FIELD

[0002] The invention relates to wireless communication and, more particularly, to coding techniques for multi-antenna transmitters.

STATEMENT REGARDING FEDERALLY SPONSORED RESEARCH OR DEVELOPMENT

[0003] This invention was made with Government support under Contract Nos. CCR-0105612, awarded by the National Science Foundation, and Contract No. DAAD19-01-2-0011 (Telcordia Technologies, Inc.) awarded by the U.S. Army. The Government may have certain rights in this invention.

BACKGROUND

[0004] By matching transmitter parameters to time varying channel conditions, adaptive modulation can increase the transmission rate considerably, which justifies its popularity for future high-rate wireless applications. The adaptive modulation makes use of channel state information (CSI) at the transmitter, which may be obtained through a feedback channel. Adaptive designs assuming perfect CSI work well only when CSI imperfections induced by channel estimation errors and/or feedback delays are limited. For example, an adaptive system with delayed error-free feedback should maintain a feedback delay $\tau \leq 0.01/f_d$, where f_d denotes the Doppler frequency. Such stringent constraint is hard to ensure in practice, unless channel fading is sufficiently slow. However, long range channel predictors relax this delay constraint considerably. An alternative approach is to account for CSI imperfections explicitly, when designing the adaptive modulator.

[0005] On the other hand, antenna diversity has been established as an effective fading counter measure for wireless applications. Due to size and cost limitations, mobile units can typically only afford one or two antennas, which motivates multiple transmit-antennas at the base station. With either perfect or partial CSI at the transmitter, the capacity and performance of multi-antenna transmissions can be further improved.

[0006] Adaptive modulation has the potential to increase the system throughput significantly by matching transmitter parameters to time-varying channel conditions. However, adaptive modulation schemes that rely on perfect channel state information (CSI) are sensitive to CSI imperfections induced by estimation errors and feedback delays.

[0007] Moreover as symbol rates increase in broadband wireless applications, the underlying Multi-Input Multi-Output (MIMO) channels exhibit strong frequency-selectivity. By transforming frequency-selective channels to an equivalent set of frequency-flat sub-channels, orthogonal frequency division multiplexing (OFDM) has emerged as an attractive transmission modality, because it comes with low-complexity (de)modulation, equalization, and decoding, to mitigate frequency-selective fading effects. One challenge for an adaptive MIMO-OFDM transmissions involves determining whether and what type of CSI can be made practically available to the transmitter in a wireless setting where fading channels are randomly varying.

SUMMARY

[0008] In general, the invention is directed to adaptive modulation schemes for multi-antenna transmissions with partial channel knowledge. The techniques are first described in reference to single-carrier, flat-fading channels. The techniques are then extended to multi-carrier, frequency-fading channels.

[0009] In particular, a transmitter is described that includes a two-dimensional beamformer where Alamouti coded data streams are power loaded and transmitted along two orthogonal basis beams. The transmitter adjusts the basis beams, the power allocation between two beams, and the signal constellation, to improve, e.g., maximize, the system throughput while maintaining a prescribed bit error rate (BER). Adaptive trellis coded modulation may also be used to further increase the transmission rate.

[0010] The described adaptive multi-antenna modulation schemes are less sensitive to channel imperfections compared to single-antenna counterparts. In order to achieve the same transmission rate, an interesting tradeoff emerges between feedback quality and hardware complexity. As an example, the rate achieved by on transmit antenna when $f_d \tau \leq 0.01$ can be provided by two transmit antennas, but with a relaxed feedback delay $f_d \tau = 0.1$, representing an order of magnitude improvement.

[0011] Next, a partial CSI model for orthogonal frequency division multiplexed (OFDM) transmissions over multi-input multi-output (MIMO) frequency selective fading channels is described. In particular, this disclosure describes an adaptive MIMO-OFDM transmitter in which the adaptive two-dimensional coder-beamformer is applied on each OFDM subcarrier, along with an adaptive power and bit loading scheme across OFDM subcarriers. By making use of the available partial CSI at the transmitter, the transmission rate may be increased or maximized while guaranteeing a prescribed error performance under the constraint of fixed transmit-power. Numerical results confirm that the adaptive two-dimensional space-time coder-beamformer (with two basis beams as the two "strongest" eigenvectors of the channel's correlation matrix perceived at the transmitter) combined with adaptive OFDM (power and bit loaded with M-ary QAM constellations) improves the transmission rate considerably.

[0012] In one embodiment, the invention is directed to a wireless communication device comprising a constellation selector, a beamformer, and a plurality of transmit antennas. The constellation selector adaptively selects a signal constellation from a set of constellations based on channel state information for a wireless communication channel, wherein the constellation selector maps information bits of an outbound data stream to symbols drawn from the selected constellation to produce a stream of symbols. The beamformer generates a plurality of coded data streams from the stream of symbols. The plurality of transmit antennas output waveforms in accordance with the plurality of coded data streams.

[0013] In another embodiment, the invention is directed to a wireless communication device comprising a plurality of adaptive modulators that each comprises: (i) a constellation selector that adaptively selects a signal constellation from a set of constellations based on channel state information for a wireless communication channel, wherein the constellation selector maps the respective information bits to symbols drawn from the selected constellation to

produce a stream of symbols, and (ii) a beamformer that generates a plurality of coded data streams from the stream of symbols. The wireless communication device further comprises a modulator to produce a multi-carrier output waveform in accordance with the plurality of coded data streams for transmission through the wireless communication channel.

[0014] In another embodiment, the invention is directed to a method comprising receiving channel state information for a wireless communication system, adaptively selecting a signal constellation from a set of constellations based on the channel state information, and coding signals for transmission by a multiple antenna transmitter based on the estimated channel information and the selected constellation.

[0015] In another embodiment, the invention is directed to a computer-readable medium comprising instructions. The instructions cause a programmable processor to receive channel state information for a wireless communication system, and select a signal constellation from a set of constellations based on the channel state information. The instructions further cause the processor to map information bits of an outbound data stream to symbols drawn from the selected constellation to produce a stream of symbols, and apply an eigen-beamformer to generate a plurality of coded data streams from the stream of symbols to produce a plurality of coded signals.

[0016] The details of one or more embodiments of the invention are set forth in the accompanying drawings and the description below. Other features, objects, and advantages of the invention will be apparent from the description and drawings, and from the claims.

BRIEF DESCRIPTION OF DRAWINGS

[0017] FIG. 1 is a graph that compares the exact bit error rates (BERs) evaluated against the approximate BERs for QAM constellations

[0018] FIG. 2 is a block diagram illustrating a wireless communication system with N_t transmit-and N_r receive-antennas.

[0019] FIG. 3 is a block diagram illustrating a two-dimensional (2D) beamformer upon which the adaptive multi-antenna transmitter described herein is based.

[0020] FIG. 4 is a graphic that plots the optimal regions for different signal constellations

[0021] FIG. 5 is a graph that plots the simulated BER and the approximate BER

[0022] FIG. 6 is a graph that plots one possible error path in adaptive trellis code modulation for 8-state trellis codes.

[0023] FIG. 7 plots the rate achieved by the adaptive transmitter.

[0024] FIG. 8 is a plot that illustrates an achieved transmission rate for a system having a single receive antenna.

[0025] FIG. 9 is a plot that illustrates a tradeoff between feedback delay and hardware complexity.

[0026] FIG. 10 is a plot that illustrates an achieved rate improvement with trellis coded modulation (TCM).

[0027] FIG. 11 is a plot that illustrates an impact of receive diversity on the adaptive TCM techniques.

[0028] FIG. 12 is a block diagram depicting an equivalent discrete-time baseband model of an OFDM wireless communication system.

[0029] FIG. 13 is plot that illustrates certain thresholds.

[0030] FIG. 14 is a plot that illustrates a power loading snapshot for certain channel realizations.

[0031] FIG. 15 is a plot illustrating certain threshold distances.

[0032] FIG. 16 is a plot illustrating a bit loading snapshot for certain channel realizations.

[0033] FIGS. 17-19 are plots that illustrate certain rate comparisons.

DETAILED DESCRIPTION

[0034] This disclosure first presents a unifying approximation to bit error rate (BER) for M-ary quadrature amplitude modulation (M-QAM). Gray mapping from bits to symbols is assumed. In order to facilitate adaptive modulation, approximate BERs, that are very simple to compute, are particularly attractive. In addition to square QAMs with $M = 2^{2i}$, rectangular QAMs with $M = 2^{2i+1}$ are considered. For exemplary purposes, the disclosure focuses on rectangular QAMs that can be implemented with two independent pulse-amplitude-modulations (PAMs): one on the In-Phase branch with size $\sqrt{2M}$, and the other on the Quadrature-phase branch with size $\sqrt{M/2}$.

[0035] Consider a non-fading channel with additive white Gaussian noise (AWGN), having variance $N_0/2$ per real and imaginary dimension. For a constellation with average energy E_s, let $d_0 := \min(|s-s'|)$ be its minimum Euclidean distance. For each constellation, define a constant g as:

$$g = \frac{3}{2(M-1)} \text{ for square } M\text{-QAM} \tag{1}$$

$$g = \frac{6}{5M-4} \text{ for rectangular } M\text{-QAM.} \tag{2}$$

The symbol energy E_s is then related to d_0^2 through the identity:

$$d_0^2 = 4g\mathrm{E}_s \tag{3}$$

The following unifying BER approximation for all QAM constellations can be adopted:

$$P_b \approx 0.2 \exp\left(-\frac{d_0^2}{4N0}\right), \tag{4}$$

which can be re-expressed as:

$$P_b \approx 0.2 \exp\left(-\frac{gEs}{N0}\right). \tag{5}$$

[0036] BPSK is a special case of rectangular QAM with $M = 2$, corresponding to $g = 1$. Hence, no special treatment is needed for BPSK. We next verify the approximate BER.

[0037] FIG. 1 is a graph that compares the exact BERs evaluated against the approximate BERs for QAM constellations with $M = 2^i$, $i \in [1,8]$. The approximation is within two dBs, for all constellations at $P_b \leq 10^{-2}$, as confirmed by FIG. 1.

[0038] FIG. 2 is a block diagram illustrating a wireless communication system with N_t transmit-and N_r receive-antennas. Focusing on flat fading channels, let $h_{\mu\nu}$ denote the channel coefficient between the μth transmit- and the νth receive- antenna, where $\mu \in [1,N_t]$ and $\nu \in [1,N_r]$. Channel coefficients may be collected in an N_t x N_r channel matrix $\mathbf{H}$ having (μ, ν)th entry $h_{\mu\nu}$. For each receive antenna ν, the channel vector $\mathbf{h}_\nu := [h_{1\nu}, \ldots, h_{N_t\nu}]^T$ is defined.

[0039] The wireless channels are slowly time-varying. The receiver obtains instantaneous channel estimates, and feeds the channel estimates back to the transmitter regularly. Based on the available channel knowledge, the transmitter adjusts its transmission to improve the performance, and increase the overall system throughput. The disclosure next specifies an exemplary channel feedback setup, and develops an adaptive multi-antenna transmission structure.

Channel Mean Feedback

[0040] For exemplary purposes, the disclosure focuses on channel mean feedback, where spatial fading channels are modeled as Gaussian random variables with non-zero mean and white covariance conditioned on the feedback. Specifically, an assumption may be adopted that transmitter x models channels x as:

$$\mathrm{H} = \overline{\mathrm{H}} + \Xi, \qquad (6)$$

where $\overline{\mathrm{H}}$ is the conditional mean of H given feedback information, and $\sim CN\left(0_{N_t \times N_r}, N_r \sigma_E^2 I_{N_t}\right)$ is the associated zero-mean error matrix. The deterministic pair $\left(\overline{\mathrm{H}}, \sigma_\epsilon^2\right)$ parameterizes the partial CSI, which is updated regularly given feedback information from the receiver.

[0041] The partial CSI parameters $\left(\overline{\mathrm{H}}, \sigma_\epsilon^2\right)$ can be provided in many different ways. For illustration purposes, a specific application scenario with delayed channel feedback is explored and used in our simulations.

[0042] With regard to delayed channel feedback, it can be assumed that: i) the channel coefficients $\{h_{\mu\nu}\}_{\mu=1,\nu=1}^{N_t N_r}$ are independent and identically distributed with Gaussian distribution $CN\left(0, \sigma_h^2\right)$; ii) the channels are slowly time varying according to Jakes' model with Doppler frequency f_d; and iii) the channels are acquired perfectly at the receiver and are fed back to the transmitter with delay τ, but without errors. Perfect channel estimation at the receiver (with infinite quantization resolution), and error-free feedback, which can be approximated by using error-free control coding and ARQ protocol in feedback channel feedback $\mathbf{H}_f$ is drawn from the same Gaussian process as H, but in τ seconds ahead of H. The corresponding entries of $\mathbf{H}_f$

and **H** are then jointly zero-mean Gaussian, with correlation coefficient $\rho := J_0(2\pi f_d \tau)$ specified from the Jakes' model, where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. For each realization of $\mathbf{H}_f$, the parameters needed in the mean feedback model of (6) are obtained as:

$$\bar{H} = E\{H \mid H_f\} = \rho H_f, \quad \sigma_E^2 = \sigma_h^2\left(1 - |\rho|^2\right) \tag{7}$$

Adaptive Two Dimensional Transmit-Beamforming

[0043] FIG. 3 is a block diagram illustrating a two-dimensional (2D) beamformer upon which the adaptive multi-antenna transmitter described herein is based. Depending on channel feedback, the information bits will be mapped to symbols drawn from a suitable constellation. The symbol stream *s(n)* will then be fed to the 2D beamformer, and transmitted through N_t antennas. The 2D beamformer uses the Alamouti code to generate two data streams $\bar{s}_1(n)$ from the original symbol stream $s(n)$ as follows:

$$\begin{bmatrix} \bar{s}_1(2n) & \bar{s}_1(2n+1) \\ \bar{s}_2(2n) & \bar{s}_2(2n+1) \end{bmatrix} = \begin{bmatrix} s(2n) & -s^*(2n+1) \\ s(2n+1) & s^*(2n) \end{bmatrix}. \tag{8}$$

The total transmission power E_s is allocated to these streams: $\delta_1 E_s$ to $\bar{s}_1(n)$, and $\delta_2 E_s = (1 - \delta_1)E_s$ to $\bar{s}_2(n)$, where $\delta_1 \in [0,1]$. Each power-loaded symbol stream is weighted by an N_t x 1 beam-steering vector $X(n) := [\chi_1(n), \ldots, \chi_{N_t}(n)]^T$ at the *n*th time slot is:

$$X(n) = \bar{s}_1(n)\sqrt{\delta_1}u_1^* + \bar{s}_2(n\sqrt{\delta_2}u_2^*) \tag{9}$$

[0044] Moving from single to multiple transmit-antennas, a number of spatial multiplexing and space time coding options are possible, at least when no CSI is available at the transmitter. An adaptive transmitter based on a 2D beamforming approach may be advantageous for a number of reasons.

[0045] For example, based on channel mean feedback, the optimal transmission strategy (in the uncoded case) is to combine beamforming (with $N_t \geq 2$ beams) with orthogonal space time block coding (STBC), where the optimality pertains to an upper-bound on the pairwise error probability, or an upper-bound on the symbol error rate. However, orthogonal STBC loses rate when $N_t > 2$, which is not appealing for adaptive modulation whose ultimate goal is

to increase the data rate given a target BER performance. On the other hand, the 2D beamformer can achieve the best possible performance when the channel feedback quality improves. Furthermore, the 2D beamformer is suboptimal only at very high SNR. In such cases, the achieved BER is already below the target, rendering further effort on BER improvement by sacrificing the rate unnecessary. In a nutshell, the 2D beamformer is preferred because of its full-rate property, and its robust performance across the practical SNR range.

[0046] In addition, the 2D beamformer structure is general enough to include existing adaptive multi-antenna approaches; e.g., the special case of $(N_t, N_r) = (2, 1)$ with perfect CSI considered. To verify this, the channels can be denoted as h_1 and h_2. Setting $(\delta_1, \delta_2) = (1,0)$, $\mathbf{u}_1 = [1,0]^T$ when $|h_1| > |h_2|$ and $\mathbf{u}_1 = [0,1]^T$ otherwise, our 2D beamformer reduces to the selective transmitter diversity (STD) scheme. Setting $(\delta_1, \delta_2) = (1,0)$ and $\mathbf{u}_1 = [h_1, h_2]^T / \sqrt{|h_1|^2 + |h_2|^2}$, our 2D beamformer reduces to the transmit adaptive array (TxAA) scheme. Finally, setting $(\delta_1, \delta_2) = (1/2, 1/2)$, $\mathbf{u}_1 = [1,0]^T$ and $\mathbf{u}_2 = [0,1]^T$ leads to the space time transmit diversity (STTD) scheme.

[0047] Moreover, due at least in part to the Alamouti structure, improved receiver processing can readily be achieved. The received symbol $y_v(n)$ on the vth antenna is:

$$\begin{aligned} y_v(n) &= x^T(n) h_v + w_v(n) \\ &= \bar{s}_1(n)\sqrt{\delta_1}\, u_1^H h_v + \bar{s}_2(n)\sqrt{\delta_2}\, u_2^H h_v + w_v(n), \end{aligned} \qquad (10)$$

where $w_v(n)$ is the additive white noise with variance $N_0/2$ per real and imaginary dimension. Eq. (10) suggests that the receiver only observes two virtual transmit antennas, transmitting $\bar{s}_1(n)$ and $\bar{s}_2(n)$, respectively. The equivalent channel coefficient from the jth virtual transmit antenna to the vth receive-antenna is $\sqrt{\delta_j}\, u_j^H h_v$ Supposing that the channels remain constant at least over two symbols, the linear maximum ratio combiner (MRC) is directly applicable to our receiver, ensuring maximum likelihood optimality. Symbol detection is performed separately for each symbol; and each symbol is equivalently passing through a scalar channel with

$$y(n) = h_{eqv} s(n) + w(n) .$$

$$h_{eqv} := \left[\delta_1 \sum_{v=1}^{N_r} \left| u_1^H h_v \right|^2 + \delta_2 \sum_{v=1}^{N_r} \left| u_2^H h_v \right|^2 \right]^{1/2}, \tag{11}$$

where $w(n)$ has variance $N_0/2$ per dimension. The transmitter influences the quality of the equivalent scalar channel h_{eqv} through the 2D beamformer adaptation of (δ_1, δ_2, $\mathbf{u}_1$, $\mathbf{u}_2$).

[0048] As yet another advantage, the combination of Alamouti's coding and transmit-beamforming may be advantages in view of emerging standards.

Adaptive Modulation Based on 2D Beamforming

[0049] Returning to FIG. 2, based on mean feedback, transmitter 4 controls eigen-beamformer x to adjust the basis beams (u_1 and u_2), the power allocation (δ_1 and δ_2), and the signal constellation of size M and energy E_s, to maximize the transmission rate while maintaining the target BER:$P_{b,target}$. For purposes of illustration, QAM constellations are adopted, N different QAM constellations with $M_i = 2^i$, where $i = 1, 2, \ldots$, N, as those exemplified above, are assumed. Correspondingly, the constellation–specific constant g can be denoted as g_i. The value of g_i is evaluated from (1), or (2), depending on the constellation M_i. When the channel experiences deep fades, the adaptive design may be allowed to suspend data transmission (this will correspond to $M_0 = 0$).

[0050] Under these assumptions, transmitter 4 perceives a random channel matrix **H** as in (6). The BER for each realization of **H** is obtained from (11) and (5) as:

$$P_b(H, M_i) \approx 0.2 \exp\left(-h_{eqv}^2 \frac{g_i E_s}{N_0} \right) \tag{12}$$

Since the realization of **H** is not available, the transmitter relies on the average BER:

$$\bar{P}_b(M_i) = E\{P_b(H, M_i)\} \approx 0.2E\left\{ \exp\left(-h_{eqv}^2 \frac{g_i E_s}{N_0} \right) \right\}, \tag{13}$$

and uses $\bar{P}_b(M_i)$ as a performance metric to select a constellation of size M_i.

Let the eigen decomposition of $\overline{H}\overline{H}^H$ be:

$$\overline{H}\overline{H}^H = U_H D_H U_H^H, \quad D_H := diag(\lambda_1, \lambda_2, \ldots, \lambda_{Nt}) \tag{14}$$

where $U_{\rm H} := \lfloor u_{\rm H,1},\ldots,u_{\rm H,N_t} \rfloor$ contains N_t eigenvectors, and D_H has the corresponding N_t eigenvalues on its diagonal in a non-increasing order $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_{N_t}$. Because $\{u_{\rm H,\mu}\}_{\mu=1}^{N_t}$ are also eigenvectors of $\overline{\rm HH}^H + {\rm N}_r\sigma_\varepsilon^2{\rm I}_{N_t}$ the correlation matrix of the perceived channel H in (6), we term them as eigen-directions, or, eigen-beams.

[0051] For any power allocation with $\delta_1 \geq \delta_2 \geq 0$ the optimal $\mathbf{u}_1$ and $\mathbf{u}_2$ minimizing $\overline{\rm P}_b({\rm M}_i)$ can be expressed as:

$$u_1 = u_{\rm H,1}, \quad u_2 = u_{\rm H,2} \tag{15}$$

In other words, the optimal basis beams for our 2D beamformer are eigen-beams corresponding to the two largest eigenvalues λ_1 and λ_2. Hereinafter, the adaptive 2D beamformer is referred to as a 2D eigen-beamformer.

Adaptive Power Allocation between Two Beams

[0052] With the optimal eigen-beams, the average BER can be obtained similarly, but with only two virtual antennas. Formally, the expected BER is:

$$\overline{\rm P}_b({\rm M}_i) \approx 0.2\prod_{\mu=1}^{2}\left[\frac{1}{1+\delta_\mu\beta_i}\exp\left(-\frac{\lambda_\mu\delta_\mu\beta_\mu}{{\rm N}_r\sigma_\varepsilon^2(1+\delta_\mu\beta_i)}\right)\right]^{{\rm N}_r} \tag{16}$$

where for notational brevity, we define

$$\beta_i := g_i\sigma_\varepsilon^2{\rm E}_s / N_0 \tag{17}$$

For a given β_i, the optimal power allocation that minimizes (16) can be found in closed-form, following derivations. Specifically, with two virtual antennas, we simplify to:

$$\delta_2 = \max(\delta_2^0, 0), \quad \delta_1 = 1 - \delta_2 \tag{18}$$

where δ_2^0 is obtained from:

$$\delta_2^0 := \frac{1+\dfrac{N_r\sigma_\varepsilon^2+\lambda_1}{(N_r\sigma_\varepsilon^2+2\lambda_1)\beta_i}}{1+\dfrac{(N_r\sigma_\varepsilon^2+2\lambda_2)(N_r\sigma_\varepsilon^2+\lambda_1)^2}{(N_r\sigma_\varepsilon^2+2\lambda_1)(N_r\sigma_\varepsilon^2+\lambda_2)^2}} - \frac{N_r\sigma_\varepsilon^2+\lambda_2}{(N_r\sigma_\varepsilon^2+2\lambda_2)\beta_i} \tag{19}$$

The optimal solution guarantees that $\delta_1 \geq \delta_2 \geq 0$; thus, more power is allocated to the stronger eigen-beam. If two eigen-beams are equally important ($\lambda_1 = \lambda_2$), the optimal solution is $\delta_1 = \delta_2 = ½$. On the other hand, if the channel feedback quality improves as $\sigma_\varepsilon^2 \to 0$, δ_1 and δ_2 are constellation dependent.

Adaptive Rate Selection with Constant Power

[0053] With perfect CSI, using the probability density function (p.d.f.) of the channel fading amplitude, the optimal rate and power allocation for single antenna transmissions has been provided. Optimal rate and power allocation for the multi-antenna transmission described herein with imperfect CSI turns out to be much more complicated. Constant power transmission can be, therefore, focused on, and only the modulation level is adjusted. Constant power transmission simplifies the transmitter design, and obviates the need for knowing the channel p.d.f..

[0054] With fixed transmission power and a given constellation, transmitter 4 computes the expected BER with optimal power splitting in two eigen-beams, per channel feedback. The transmitter then chooses the rate-maximizing constellation, while maintaining the target BER. Since the BER performance decreases monotonically with the constellation size, the transmitter finds the optimal constellation to be:

$$M = \arg\max \quad \bar{\mathrm{P}}_b(M) \leq \mathrm{P}_{b,target} \tag{20}$$

$$M \mathrm{E}\{M_i\}_{i=0}^{N}$$

This equation can be solved by trial and error; starting with the largest constellation $M_i = M_N$, and then decreasing i until the optimal M_i is found.

[0055] Although there are $N_t N_r$ entries in $\mathbf{H}$, constellation selection depends only on the first two eigen-values λ_1 and λ_2. The two dimensional space of (λ_1,λ_2) can be split in $N+1$ disjoint regions $\{D_i\}_{i=0}^{N}$ each associated with one constellation. Specifically,

$$M = M_i, \text{ when } (\lambda_1,\lambda_2) \in D_i, \forall i = 0,1,\ldots,N \tag{21}$$

can be chosen. The rate achieved by system 2 of FIG. 2 is therefore

$$R = \sum_{i=1}^{N} \log_2(M_i) \iint_{D_i} p(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2, \tag{22}$$

where $p(\lambda_1, \lambda_2)$ is the joint p.d.f. of λ_1 and λ_2. The outage probability is thus:

$$\mathrm{P}_{out} = \iint_{D_0} p(\lambda_1, \lambda_2) d\lambda_1 d\lambda_2 \, . \tag{23}$$

The fading regions can be specified. Since $\lambda_2 = \lambda_1$, we have $a := \lambda_2 / \lambda_1 \in [0,1]$

To specify the region D_i in the (λ_1, λ_2) space, the intersection of D_i with each straight line can be specified as $\lambda_2 = a\lambda_1$ where $a \in [0,1]$. Specifically, the fading region D_i on each line will reduce to an interval. This interval on the line $\lambda_2 = a\lambda_1$ will be denoted as $[\alpha_i(\alpha), \alpha + 1(\alpha))$, during which the constellation M_i is chosen. In addition, $\alpha_0(\alpha) = 0$ and $\alpha_{N+1}(a) = \infty$. The boundary points $\{\alpha_i(\alpha)\}_{i=1}^{N}$ remain to be specified.

[0056] For a given constellation M_i and power allocation factors $(\delta_1, \delta_2 = 1 - \delta_1)$ the minimum value of λ_1 on the line of $\lambda_2 = a\lambda_1$ can be determined so that $\overline{\mathrm{P}}_b(M_i) \leq \mathrm{P}_{b,target}$ as:

$$\lambda_1(a, \delta_1 M_i) = \sigma_\varepsilon^2 \left(\frac{\delta_1 \beta_i}{1 + \delta_1 \beta_i} + \frac{a \delta_2 \beta_i}{1 + \delta_2 \beta_i} \right)^{-1}$$

$$\times \ln \left(\frac{0.2}{\mathrm{P}_{b,target} \left[(1 + \delta_1 \beta_i)(1 + \delta_2 \beta_i) \right]^{N_r}} \right) \tag{24}$$

Since the optimal $\delta_1 \in [1/2, 1]$ will lead to the minimal λ_1 that satisfies the BER requirement, the boundary point $\alpha_i(a)$ can be found as:

$$\alpha_i(a) = \min_{\delta_1 \in [1/2,1]} \lambda_1(a, \delta_1, M_i) \tag{25}$$

The minimization is a one-dimensional search, and it can be carried out numerically. Having specified the boundaries on each line, the fading regions associated with each constellation in the two dimensional space can be plotted, as illustrate in further detail below.

[0057] In the general multi-input multi-output (MIMO) case, each constellation M_i is associated with a fading region D_i on the two dimensional plane (λ_1, λ_2). Several special cases exist, where the fading region is effectively determined by fading intervals on the first

eigenvalue λ_1. In such cases, the boundary points are denoted as $\{\bar{\alpha}_i\}_{i=0}^{N+1}$. The constellation M_i is chosen when $\lambda_1 \in [\bar{\alpha}_i, \bar{\alpha}_{i+1})$ The following may then be obtained:

$$R = \sum_{i=1}^{N} \log_2(M_i) \int_{\bar{\alpha}_i}^{\bar{\alpha}_{i+1}} p(\lambda_1) d\lambda_1$$

$$= \sum_{i=1}^{N} \log_2(M_i) [F(\bar{\alpha}_{i+1}) - F(\bar{\alpha}_i)] \tag{26}$$

where $F(\chi) := \int_0^{\chi} p(\lambda_1) d\lambda_1$ is the cumulative distribution function (c.d.f.) of λ_1. The outage becomes:

$$\mathrm{P}_{out} = F(\bar{\alpha}_1) \tag{27}$$

To calculate the rate and outage, it suffices to determine the p.d.f. of λ_1, and the boundaries $\{\bar{\alpha}_i\}_{i=1}^{N}$. For multiple transmit – and a single receive – antennas, $N_r = 1$, and there is only one non-zero eigen-value λ_1, and thus $a = \lambda_2/\lambda_1 = 0$. The boundary points are:

$$\bar{\alpha}_i = \alpha_i(0) \quad \forall i = 0,1,\ldots,N \tag{28}$$

where $\alpha_i(a)$ is specified in (25).

[0058] When $N_r = 1$, the channel h_1 is distributed as $CN(0, \mathrm{I}_{N_t})$. With delayed feedback considered in Example 2, we have $\lambda_1 = (|\rho|^2)\|h_1\|^2 = |\rho|^2 \sum_{\mu=1}^{N_t} |h_{\mu 1}|^2$ which is Gamma distributed with parameter N_t and mean $\mathrm{E}\{\lambda_1\} = |\rho|^2 N_t$ The p.d.f. and c.d.f. of λ_1 are:

$$p(\lambda_1) = \left(\frac{1}{|\rho|^2}\right)^{N_t} \frac{\lambda_1^{N_t - 1}}{(N_t - 1)!} \exp\left(-\frac{\lambda_1}{|\rho|^2}\right), \lambda_1 \geq 0 \tag{29}$$

$$F(\chi) = \int_0^{\chi} p(\lambda_1) d\lambda_1$$

$$= 1 - e^{-\chi/|\rho|^2} \sum_{j=0}^{N_t - 1} \frac{1}{j!} \left(\frac{\chi}{|\rho|^2}\right)^j, \chi \geq 0 \tag{30}$$

Plugging (30) and (28) into (26), the rate becomes readily available.

[0059] Turning to the MIMO case, the adaptive 2D beamformer described herein subsumes a 1D beamformer by setting $\delta_1 = 1$ and $\delta_2 = 0$. Numerical search is now unnecessary, and $\delta_2 = 0$ does not depend on a anymore. The following can be simplified:

$$\begin{aligned}\bar{\alpha}_i &= \lambda_1(a,1,M_i) \\ &= \frac{\sigma_\epsilon^2}{\beta_i}(1+\beta_i)\ln\left(\frac{0.2}{\mathrm{P}_{b,target}(1+\beta_i)^{N_r}}\right)\end{aligned} \tag{31}$$

The fading region thus depends only on λ_1.

[0060] FIG. 4 is a graphic that plots the optimal regions for different signal constellations with $\mathrm{P}_b = 10^{-3}, \mathrm{E}_s / N_0 = 15dB$ and $\rho = 0.9$. As the constellation size increases, the difference between 1D and 2D beamforming decreases.

[0061] With perfect CSI $\left(\sigma_\epsilon^2 = 0.\bar{\mathrm{H}} = \mathrm{H}\right)$ the optimal loading ends up being $\delta_1 = 1, \delta_2 = 0$. Therefore, the optimal transmission strategy in this case is 1D eigen-beamforming. The results apply to 1D beamforming, but with $\sigma_\epsilon^2 = 0$ Specifically, we simplify to

$$\mathrm{P}_b(M_i) \approx 0.2\exp\left(-\lambda_1 \frac{g_i \mathrm{E}_s}{N_0}\right) \tag{32}$$

and to

$$\bar{\alpha}_i = \lambda_1(a,1,M_1) = \frac{1}{g_i \mathrm{E}_s / N_0} \ln\left(\frac{0.2}{\mathrm{P}_{b,target}}\right). \tag{33}$$

Eq. (32) reveals that the MIMO antenna gain is introduced solely through λ_1, the maximum eigenvalue of (or, HH^{H}).

[0062] Notice that with *perfect* CSI, one can enhance spectral efficiency by adaptively transmitting parallel data streams over as many as N_t eigen-channels of. These data streams can be decoded separately at the receiver. However, this scheme can not be applied when the available CSI is imperfect, since the eigen-directions of $\overline{\mathrm{HH}}^{\mathrm{H}}$ are no longer the eigen-directions of the true channel HH^{H}. As a result, these parallel streams will be coupled at the receiver side, and will interfere with each other. This coupling calls for higher receiver

complexity to perform joint detection, and also complicates the transmitter design, since no approximate BER expressions are readily available.

Adaptive Trellis Coded Modulation

[0063] Next, coded modulation is considered. Recall that each information symbol *s(n)* is equivalently passing through a scalar channel in the proposed transmitter. Thus, conventional channel coding can be applied. For exemplary purpose, trellis coded modulation (TCM) is focused on, where a fixed trellis code is superimposed on uncoded adaptive modulation for fading channels. The single antenna design with perfect CSI can be extended to the MIMO system described herein with partial, i.e., imperfect, CSI.

[0064] For adaptive trellis coded modulation, out of n information bits, k bits pass through a trellis encoder to generate $k + r$ coded bits. A constellation of size 2^{n+r} is partitioned into 2^{k+r} subsets with size 2^{n-k} each. The $k + r$ coded bits specify which subset to be used, and the remaining n-k uncoded bits specify one signal point from the subset to be transmitted. The trellis code may be fixed, and the signal constellation may be adapted according to channel conditions. Different from the uncoded case, the minimum constellation size now is 2^{k+r} with each subset containing only one point. With a constellation of size M_i, only $\log2(M_i) - r$ bits are transmitted.

BER approximation for AWGN channels

[0065] Let d_{free} denote the minimum Euclidean distance between any pair of valid codewords. At high SNR, the error probability resulting from nearest neighbor codewords dominates. The dominant error events have probability:

$$\begin{aligned} \mathrm{P_E} &\approx N(d_{free})Q\left(\sqrt{\frac{d_{free}^2}{2N_0}}\right) \qquad (34) \\ &\approx 0.5N(d_{free})\exp\left(-\frac{d_{free}^2}{4N_0}\right) \end{aligned}$$

where $N(d_{free})$ is the number of nearest neighbor codewords with Euclidean distance d_{free}. Along with (4) for the uncoded case, the BER can be approximated by:

$$P_{b,TCM} \approx c_2 P_E \approx c_3 \exp\left(-\frac{d_{free}^2}{4N_0}\right) \quad (35)$$

where the constants c_2 and c_3 need to be determined. For each chosen trellis code, one constant c_3 may be used for all possible constellations to facilitate the adaptive modulation process.

[0066] For each chosen trellis code and signal constellation M_i, the ratio of d_{free}^2 / d_0^2 is fixed. For each prescribed trellis code, we define:

$$g_i' = \frac{d_{free}^2}{d_0^2} g_i \text{, for the constellation } M_i. \quad (36)$$

Substituting (36) and (3) into (35), the approximate BER for constellation M_i can be obtained as:

$$P_{b,TCM}(M_i) \approx c_3 \exp\left(-\frac{g_i' E_s}{N_0}\right) \quad (37)$$

The four-state trellis code can be checked with $k = r = 1$. The constellations of size $M_i = 2^i, \forall i \in [2,8]$ are divided into four subsets, following the set partitioning procedure. Let d_j denote the minimum distance after the jth set partitioning. For QAM constellations, we have $d_{j+1} / d_j = \sqrt{2}$. When $M > 4$, parallel transitions dominate with $d_{free}^2 = d_2^2 = 4d_0^2$. With $M = 4$, no parallel transition exists, and we have $d_{free}^2 = d_0^2 + 2d_1^2 = 5d_0^2$. We find the parameter $c_3 = 1.5 = 0.375\ N(d_{free})$ for the four-state trellis, where $N(d_{free}) = 4$.

[0067] FIG. 5 is a graph that plots the simulated BER and the approximate BER in (37). The approximation is within 2dB for BER less than 10^{-1}.

[0068] FIG. 6 is a graph that plots the trellis for the eight-state trellis code, which may also be checked with $k = 2$ and $r = 1$. The constellations of size $M = 2^i$, $\forall i \varepsilon$ are divided into eight subsets. The subset sequences dominate the error performance with $d_{free}^2 = d_0^2 + sd_1^2 = 5d_0^2$ for all constellations. We choose $c_3 = 6 = 0.375N(d_{free})$ for the eight-state trellis code, where $N(d_{free}) = 16$. The approximation is within 2dB for BER less than 10^{-1}.

Adaptive TCM for fading channels

[0069] The adaptive coded modulation with mean feedback may now be specified. Since the transmitted symbols are correlated in time, a time index t is explicitly associated for each variable e.g., $\mathbf{H}(t)$ is used to denote the channel perceived at time t. The following average error probability at time t can be calculated based on (11) and (37):

$$\bar{P}_{b,TCM}(M_i,t) = E\{P_{b,TCM}(H(t),M_i)\}$$

$$\approx c_3 E\left\{\exp\left(-h_{eqv}^2(t)\frac{g_i' E_s}{N_0}\right)\right\}. \tag{38}$$

At each time t when updated feedback arrives, transmitter 4 automatically selects the constellation:

$$M(t) = \arg\max_{M\in\{M_i\}_{i=k+r}^{N}} \bar{P}_{b,TCM}(M,t) \le P_{b,target} \tag{39}$$

By the similarity of (37) and (5), we end up with an uncoded problem with constellation M_i having a modified constant g_i' and conveying $\log_2(M_i) - r$ bits.

[0070] However, distinct from uncoded modulation, the coded transmitted symbols are correlated in time. Suppose that the channel feedback is frequent. The subset sequences may span multiple feedback updates, and thus different portions of one subset sequence may use subsets partitioned from different constellations. The transmitter design in (39) implicitly assumes that all dominating error events are confined within one feedback interval. Nevertheless, this design guarantees the target BER for all possible scenarios. Since the dominating error events may occur between parallel transitions, or between subset sequences, this disclosure explores all of the possibilities:

1) <u>Parallel transitions dominate:</u> The parallel transitions occur in one symbol interval, and thus depend only on one constellation selection. The transmitter adaptation in (39) is in effect.

2) <u>Subset sequences dominate:</u> The dominating error events may be limited to one feedback interval, or, may span multiple feedback intervals. If the dominating error events are within one feedback interval, the transmitter adaptation in (39) is certainly effective. On the other hand, the error path may span multiple feedback intervals, with different portions of the error path using subsets partitioned from different constellations.

[0071] We focus on any pair of subset sequences c_1 and c_2. For brevity, it is assumed that the error path spans two feedback intervals (or updates), at time t_1 and t_2. Different constellations are chosen at time t_1 and t_2, resulting in different $d_0^2(t_1)$ and $d_0^2(t_2)$ As illustrated in FIG. 6, the distance between c_1 and c_2 can be partitioned as: $d^2(c_1,c_2|t_1,t_2)=d^2(t_1)+d^2(t_2)$ The contribution of $d^2(t_1)$at time t_1 is the minimum distance between subsets $\zeta_0(t_1)$ and $\zeta_2(t_1)$ plus the minimum distance between subsets $\zeta_0(t_1)$and $\zeta_3(t_1), i.e., d^2(t_1)=d_1^2(t_1)+d_0^2(t_1)=3d_0^2(t_1)$. Similarly, we have $d^2(t_2)=d_1^2(t_2)=2d_0^2(t_2)$

[0072] Now, two virtual events can be constructed that the error path between c_1 and c_2 experiences only on feedback: One at t_1 and the other at t_2. For j = 1,2, the average pairwise error probability is defined as:

$$\bar{\mathrm{P}}(c_1 \to c_2 \big| t_j) = 0.5\mathrm{E}\left\{\exp\left(-\frac{h_{eqv}^2(t_j)d^2(c_1,c_2|t_j)}{}\right)\right\} \tag{40}$$

Next, the following constants are defined:

$$b_1 := \frac{\tilde{d}(t_1)}{d^2(c_1,c_2|t_1)'} \quad b_2 := \frac{\tilde{d}(t_2)}{d^2(c_1,c_2|t_2)} \tag{41}$$

It is clear that $b_1 + b_2 = 1$, and $0 < b_1, b_2 \leq 1$.

[0073] When the error path between c1 and c2 spans multiple feedback intervals, the average PEP decreases relative to the case of one feedback interval. Since the conditional channels at different times are independent,

$$\begin{aligned}
&\mathrm{E}\{\mathrm{P}(c_1 \to c_2|t_1,t_2)\} \\
&= 0.5\mathrm{E}\left\{\exp\left(-\frac{h_{eqv}^2(t_1)\tilde{d}^2(t_1)}{4N_0}\right)\right\} \\
&\times \mathrm{E}\left\{\exp\left(\frac{-h_{eqv}^2(t_2)\tilde{d}^2(t_2)}{4N_0}\right)\right\} \\
&\leq 0.5\left[\frac{\bar{\mathrm{P}}(c_1 \to c_2|t_1)}{0.5}\right]^{b_1}\left[\frac{\bar{\mathrm{P}}(c_1 \to c_2|t_2)}{0.5}\right]^{b_2} \\
&\leq \max\left(\bar{\mathrm{P}}(c_1 \to c_2|t_1), \bar{\mathrm{P}}(c_1 \to c_2|t_2)\right)
\end{aligned} \tag{42}$$

where in deriving (42), the inequality in (47) (proved below) is used. Eq. (42) reveals that the worst case happens when the error path between subset sequences spans only on feedback. In such cases, however, we have guaranteed the average BER in (39), for each of the feedback intervals, the average pairwise error probability decreases, and thus the average BER (proportional to the dominating pairwise error probability is approximated in (35)) is guaranteed to stay below the target.

[0074] In summary, the transmitter adaptation in (39) guarantees the prescribed BER. With perfect CSI, this adaptation reduces to a point where d_0 is maintained for each constellation choice. The techniques described herein are simpler in comparison to some conventional approaches in the sense that the described techniques do not need to check all distances between each pair of subsets.

Examples

[0075] In simulation purposes, the channel setup is adopted with $\sigma_h^2 = 1$. Recall that the feedback quality σ_ε^2 is related to the correlation coefficient $J_0(2\pi f_d \tau)$ via $\sigma_\varepsilon^2 = 1 - |\rho|^2$.. With $\rho = 0.95, 0.9, 0.8$, we have $\sigma_\varepsilon^2 = -10.1, -7.2, -4,4dB$. For fair comparison among different setups, the average received SNR is used in all plots and defined as:

$$averageSNR := (1 - \mathrm{P}_{out})\mathrm{E}_s / N_0 \qquad (43)$$

[0076] FIG. 7 plots the rate achieved by the adaptive transmitter 4 with $P_{b,target} = 10^{-3}$, $N_t = 2$, $N_r = 1$, and $\rho = 1, 0.95, 0.9, 0.8, 0$. As illustrated in FIG. 7, it is clear that the rate decreases relatively fast as the feedback quality drops.

[0077] For comparison, FIG. 7 also plots the channel capacity with mean feedback, using the semi-analytical result. As shown in FIG. 7, the capacity is less sensitive to channel imperfections. The capacity with perfect CSI is larger than the capacity with no CSI by about $log_2(N_t) = 1$ bit at high SNR, as predicted. With $\rho = 0.9$, the adaptive uncoded modulation is about 11dB away from capacity.

[0078] FIG. 8 is a plot that illustrates the achieved transmission rate with Nr = 1, $P_{b,target} = 10^{-3}$, and $\rho = 0.9$. As shown in FIG. 8, the achieved transmission rate increases as the number of transmit antennas increases. The largest rate improvement occurs when N_t increases from one to two.

[0079] FIG. 9 is a plot that illustrates the tradeoff between feedback delay and hardware complexity. As illustrated, one tradeoff value is $f_d T = 0.01$ for single antenna transmissions. FIG. 9, verifies that with two transmit antennas, the achieved rate with $f_d T = 0.1$ ($\rho = 0.904$) coincides with that corresponding to one transmit antenna with perfect CSI ($f_d T \leq 0.01$); hence, more than ten times of feedback delay can be tolerated. The rate with $N_t = 4$ and $f_d T = 0.16$ ($p = 0.76$) is even better than that of $N_t = 1$ with perfect CSI. To achieve the same rate, the delay constraint with single antenna can be relaxed considerably by using more transit antennas, an interesting tradeoff between feedback quality and hardware complexity. FIG. 9 also reveals that the adaptive deign becomes less sensitive to CSI imperfections, when the number of transmit antenna increases.

[0080] FIG. 10 is a plot that illustrates the achieved rate improvement with trellis coded modulation. In this example, the four-state and eight-state trellis codes described above were tested. First $P_{b,target}$ was set to10^{-6}, $N_t = 2$, $N_r = 1$. When the feedback quality is near perfect (p = 0.99), the rate is considerably increased by using trellis coded modulation instead of uncoded modulation, in agreement with the prefect CSI case. However, the achieved SNR gain decreases quickly as the feedback quality drops, as shown in FIG. 10. This can be predicted, since increasing the Euclidean distance by TCM with set partitioning is less effective for fading channels ($\rho < 1$) than for AWGN channels ($\rho = 1$). If affordable, coded bits can be interleaved to benefit from time diversity, as suggested. This is suitable for the 8-state TCM, where the subset sequences dominate the error performance.

[0081] On the other hand, the Euclidena distance becomes the appropriate performance measure, when the number of receive antennas increases, as established. The SNR gain introduced by TCM is thus restored, as shown in Fig. 11 with $N_r = 2, 4$.

[0082] Comparing FIG. 10 with FIG. 7, one can observe that the adaptive system is more sensitive to noisy feedback when the prescribed bit error rate is small (10^{-6}) as opposed to large (10^{-3}).

[0083] In accordance with these techniques, adaptive modulation for multi-antenna transmissions with channel mean feedback can be achieved. Based on a two dimensional beamformer, the proposed transmitter optimally adapts the basis beams, the power allocation between two beams, and the signal constellation, to maximize the transmission rate while guaranteeing a target BER. Both uncoded and trellis coded modulation have been addressed.

Numerical results demonstrated the rate improvement enabled by adaptive multi-antenna modulation, and pointed out an interesting tradeoff between feedback quality and hardware complexity. The proposed adaptive modulation maintains low receiver complexity thanks to the Alamouti structure.

Adaptive Orthogonal Frequency Division (OFDM) Multiplexed Transmissions

[0084] The techniques described above for adaptive modulation over MIMO *flat-fading* channels are hereinafter extended to adaptive MIMO-OFDM transmissions over *frequency-selective* fading channels based on partial CSI. As further described below, an OFDM transmitter applies the adaptive two-dimensional space-time coder-beamformer on each OFDM subcarrier, with the power and bits adaptively loaded across subcarriers, to maximize transmission rate under performance and power constraints.

[0085] This problem is challenging because information bits and power should be optimally allocated over space *and* frequency, but its solution is equally rewarding because high-performance high-rate transmissions can be enabled over MIMO frequency-selective channels. As further described, the techniques include:

- Quantification of partial CSI for frequency selective MIMO channels, and formulation of a constrained optimization problem with the goal of maximizing rate for a given power budget, and a prescribed BER performance.
- Design of an optimal MIMO-OFDM transmitter as a concatenation of an adaptive modulator, and an adaptive two-dimensional coder-beamformer.
- Identification of a suitable threshold metric that encapsulates the allowable power and bit combinations, and enables joint optimization of the adaptive modulator-beamformer.
- Incorporation of algorithms for joint power and bit loading across MIMO-OFDM subcarriers, based on partial CSI.
- Illustration of the tradeoffs emerging among rate, complexity, and the reliability of partial CSI, using simulated examples.

[0086] FIG. 12 is a block diagram of a wireless communication system 30 in which an adaptive MIMO-OFDM transmitter 32 applies adaptive two-dimensional coder-beamformers 34A-34N across each OFDM subcarrier, along with an adaptive power and bit loading

scheme. In particular, FIG. 12 depicts an equivalent discrete-time baseband model of an OFDM wireless communication system 30 equipped with K subcarriers, N_t transmit-, and N_r receive-antennas, signaling over a MIMO frequency selective fading channel. Per OFDM sub-carrier, transmitter 32 deploys one of adaptive two-dimensional (2D) coder-beamformers 34A-34N. Each of 2D coder-beamers 34 combines Alamouti's space time block coding (STBS) with transmit beamforming. Higher-dimensional coder-beamformers based on orthogonal STBS with $N_t > 2$, can be also applied, as detailed below. However, the 2D coder-beamformers 34 strike desirable performance-rate-complexity tradeoffs, and for this reason, the 2D case is illustrated for exemplary purposes.

[0087] To apply the 2D coder-beamformer per subcarrier, two consecutive OFDM symbols are paired to form on space-time coded OFDM block. Due to frequency selectivity, different subcarriers experience generally different channel attenuation. Hence, in addition to adapting the 2D coder-beamformer on each subcarrier, the total transmit-power may also be judiciously allocated to different subcarriers based on the available CSI at transmitter 32.

[0088] Let n be used to index space time coded OFDM blocks (pairs of OFDM symbols), and let k denote the subcarrier index; i.e., $k \in \{0,1,\ldots,K-1\}$. Let $P[n;k]$ stand for the power allocated to the kth subcarrier of the nth block. Then, depending on $P[n;k]$, a constellation (alphabet) $A[n;k]$ consisting of $M[n;k]$ constellation points is selected. In addition to square QAMs with $M[n;k] = 2^{2i}$, that have been used extensively in adaptive modulation, rectangular QAMs with $M[n;k] = 2^{2i+1}$ are also considered. Similar to the previous analysis, the subsequent analysis focuses on rectangular QAMs that can be implemented with two independent PAMs: one for the In-phase branch with size $\sqrt{2M[n;k]}$ and the other for the Quadrature-phase branch with size $\sqrt{M[n:k]/2}$ as those studied. Due to the independence between I-Q branches, this type of rectangular QAM incurs modulation and demodulation complexity similar to square QAM.

[0089] For each block time-slot n, the input to each of 2D coder-beamformer 34 used per subcarrier entails two information symbols, $s_1[n;k]$ and $s_2[n;k]$, drawn from $A[n;k]$, with each one conveying

$$b[n;k] = \log_2(M[n;k]) \tag{44}$$

bits of information. These two information symbols will be space-time coded, power-loaded, and multiplexed by the 2D beamformer to generate an $N_t \times 2$ space-time (ST) matrix as:

$$X[n;k] = \underbrace{[u_1^*[n;k], u_2^*[n;k]]}_{:=U^*[n;k]} \cdot \begin{bmatrix} \sqrt{\delta_1[n;k]} & 0 \\ 0 & \sqrt{\delta_2[n;k]} \end{bmatrix} \cdot \begin{bmatrix} s_1[n;k] & -s_2^*[n;k] \\ s_2[n;k] & s_1^*[n;k] \end{bmatrix}, \tag{45}$$

where **S**[*n;k*] is the well-known Alamouti ST code matrix; **U**[*n;k*] is the multiplexing matrix formed by two $N_t \times 1$ basis-beam vectors $\mathbf{u}_1[n;k]$ and $\mathbf{u}_2[n;k]$; and **D**[*n;k*] is the corresponding power allocation matrix on these two basis-beams with $0 \le \delta_1[n;k], \delta_2[n;k] \le 1$, and $\delta_1[n;k] + \delta_2[n;k] = 1$. In the two time slots corresponding to the two OFDM symbols involved in the *n*th ST coded block, the two columns of **X**[*n;k*] are transmitted on the *k*th subcarrier over N_t transmit-antennas.

[0090] For purposes of illustration, it is assumed that the MIMO channel is invariant during each space-time coded block, but is allowed to vary form block to block. Let $h_{\mu,\nu}[n] := [h_{\mu,\nu}[n;0], \ldots, h_{\mu,\nu}[n;L]]^T$ be the baseband equivalent FIR channel between the μth transmit- and the νth receive-antenna during the *n*th block, where $1 \le \mu \le N_t$, $1 \le \nu \le N_r$, and L is the maximum channel order of all $N_t N_r$ channels. With $f_k := [1, e^{j2\pi k/N}, \ldots, e^{j2\pi kL/N}]^T$ the frequency response of $\mathbf{h}_{\mu\nu}[n]$ on the *k*th subcarrier is:

$$H_{\mu,\nu}[n;k] = \sum_{l=0}^{L} h_{\mu\nu}[n;l] e^{-j2\pi kl/N} = f_k^H h_{\mu\nu}[n] \tag{46}$$

[0091] Let **H**[*n;k*] be the $N_t \times N_r$ matrix having $H_{\mu\nu}[n;k]$ as its (μ, ν)th entry. To isolate the transmitter design from channel estimation issues at the receiver, we suppose that the receiver has perfect knowledge of the channel $H[n;k], \forall n, k$.

[0092] With **Y**[*n;k*] denoting the *n*th received block on the *k*th subcarrier, we can express the input-output relationship per subcarrier and ST coded OFDM block as

$$\begin{aligned} Y[n;k] &= H^T[n;k] X[n;k] + W[n;k] \\ &= H^T[n;k] U^*[n;k] D[n;k] S[n;k] + W[n;k] \end{aligned} \tag{47}$$

where $\mathbf{W}[n;k]$ stands for the additive white Gaussian noise (AWGN) at the receiver with each entry having variance $N_0/2$ per real and imaginary dimension. Based on (47), one can view our coded-beamformed MIMO OFDM transmissions per subcarrier as an Alamouti transmission with ST matrix $\mathbf{S}[n;k]$ passing through an equivalent channel matrix $\mathbf{B}^T[n;k] := \mathbf{H}^T[n;k]\ \mathbf{U}^*[n;k]\ \mathbf{D}[n;k]$. With knowledge of this equivalent channel and maximum ratio combining (MRC) at receiver 38, it can be verified that each information symbol is thus passing through an equivalent scalar channel with I/O relationship

$$z_i[n;k] = h_{eqv}[n;k]s_i[n;k] + w_i[n;k], i = 1,2, \tag{48}$$

where the equivalent channel is:

$$h_{eqv}[n;k] = \|B[n;k]\|_F = \left[\delta_1[n;k]\|H^H[n;k]u_1[n;k]\|_F^2 + \delta_2[n;k]\|H^H[n;k]u_2[n;k]\|_F^2\right]^{1/2}. \tag{49}$$

Partial CSI for Frequency-Selective MIMO Channels

[0093] Mean feedback has been described above in reference to flat-fading multi-antenna channels to account for channel uncertainty at the transmitter, where the fading channels are modeled as Gaussian random variables with non-zero mean and white covariance. This mean feedback model is adopted for each OFDM subcarrier of the OFDM system 30 of FIG. 12. Specifically, it is assumed that on each subcarrier k, transmitter 32 obtains an un-biased channel estimate $\overline{\mathrm{H}}[n;k]$ either through a feedback channel, or during a duplex mode operation, or, by predicting the channel from past blocks. Transmitter 32 treats this "nominal channel" $\overline{\mathrm{H}}[n;k]$ as deterministic, and in order to account for CSI uncertainty, it adds a "perturbation" term. The partial CSI of the true $N_t \times N_r$ MIMO channel $\mathbf{H}[n;k]$ at transmitter 32 is thus perceived as:

$$\breve{H}[n;k] = \overline{H}[n;k] + \Xi[n;k], k = 0,1,\ldots,K-1, \tag{50}$$

where $\Xi[n;k]$ is a random matrix Gaussian distributed according to

$CN(0_{N_t x N_r}, N_r\sigma_\varepsilon^2[n;k]I_{N_t})$ The variance $\sigma_\varepsilon^2[n;k]$ encapsulates the CSI reliability on the kth subcarrier.

[0094] Suppose that the FIR channel taps have been acquired perfectly at the receiver, and are fed back to the transmitter with a certain delay, but without errors thanks to powerful

error control codes used in the feedback. Let us also assume that the following conditions hold true:

i) The $L + 1$ taps $\{h_{\mu\nu}[n;l]\}_{l=0}^{L}$ in $h_{\mu\nu}[n]$ are uncorrelated, but not necessarily identically distributed (to account for e.g., exponentially decaying power profiles). Each tap is zero-mean Gaussian with variance $\sigma_{\mu\nu}^{2}[l]$ Hence,

$$h_{\mu\nu}[n] \sim CN\left(0, \sum{}_{\mu\nu}\right), where \sum{}_{\mu\nu} := diag\left(\sigma_{\mu\nu}^{2}[0], \ldots, \sigma_{\mu\nu}^{2}[l]\right)$$

ii) The FIR channels $\{h_{\mu\nu}[n]\}_{\mu=1,\nu=1}^{N_t,N_r}$ between different transmit- and receive-antenna pairs are independent. This requires antennas to be spaced sufficiently far apart from each other.

iii) All FIR channels have the same total energy on the average $\sigma_{h}^{2} = tr\{\sum{}_{\mu\nu}\}, \forall \mu, \nu$. This is reasonable in practice, since the multi-antenna transmissions experience the same scattering environment.

iv) All channel taps are time varying according to Jakes' model with Doppler frequency f_d.

[0095] At the nth block, assume the channel feedback $\{h_{\mu\nu}^{f}[n]\}_{\mu=1,\nu=1}^{N_t,N_r}$, that corresponds to the true channels N_b blocks earlier is obtained; i.e. $h_{\mu\nu}^{f}[n] = h_{\mu\nu}[n - N_b]$. Assume each space time coded block has time duration T_b seconds. Then, $h_{\mu\nu}^{f}[n]$ is drawn from the same Gaussian distribution as $h_{\mu\nu}[n]$, but N_bT_b seconds ahead. Let $\rho := J_0(2\pi f_d N_b T_b)$ denote the correlation coefficient specified by Jakes' model, where $J_0(\cdot)$ is the zeroth order Bessel function of the first kind. The MMSE predictor of $h_{\mu\nu}[n]$, and i), is $\bar{h}_{\mu\nu}[n] = \rho_h\, i_{\mu\nu}^{f}[n]$. To account for the prediction imperfections, the transmitter forms an estimate $h_{\mu\nu}[n]$ as:

$$\breve{h}_{\mu\nu}[n] = \bar{h}_{\mu\nu}[n] + \xi_{\mu\nu}[n], \tag{51}$$

where $\xi_{\mu\nu}[n]$ is the prediction error. Under i), it can be verified that

$$\xi_{\mu\nu}[n] \sim CN\left(0, \left(1 - |\rho|^2\right) \sum{}_{\mu\nu}\right). \tag{52}$$

[0096] The mean feedback model on channel taps described above can be translated to the CSI on the channel frequency response per subcarrier. Based on this, the matrices with ($\mu\nu$)th entries can be obtained:

$\left[\breve{H}[n;k]\right]_{\mu\nu} = f_k^H \breve{h}_{\mu\nu}[n], \left[\bar{H}[n;k]\right]_{\mu\nu} = f_k^H \bar{h}_{\eta\nu}, and \left[\Xi[n;k]\right]_{\mu\nu} = f_k^H \xi_{\mu\nu}[n]$ Using i), ii), and (52), it can be verified that $\Xi[n;k]$ has covariance matrix $N_r\left(1-|\rho|^2\right)\sigma_h^2 I_{N_t}$. Notice that in this case, the uncertainty indicators $\sigma_\varepsilon^2[n;k] = \left(1-|\rho|^2\right)\sigma_h^2$ are common to all subcarriers.

[0097] Notwithstanding, the partial CSI has also unifying value. When $K = 1$, it boils down to the partial CSI for flat fading channels. With $\sigma_\varepsilon^2 = 0$, it reduces to the perfect CSI of the MIMO setup considered. When $N_t = N_r = 1$, it simplifies to the partial CSI feedback used for SISO FIR channels. Furthermore, with $N_t = N_r = 1$ and $\sigma_\varepsilon^2 = 0$ it is analogous to perfect CSI feedback for wireline DMT channels.

[0098] One objective is to optimize the MIMO-OFDM transmissions in FIG. 12, based on partial CSI available at the transmitter. Specifically, we may want to maximize the transmission rate subject to a power constraint, while maintaining a target BER performance on each subcarrier. Let $\overline{BER}[n;k]$ denote the perceived average BER at the transmitter on the kth subcarrier of the nth block, and $\overline{BER}_0[k]$ stand for the prescribed target BER on the kth subcarrier. The target BERs can be identical, or, different across subcarriers, depending on system specifications. Recall that each space-time coded block conveys two symbols, $s_1[n;k], s_2[n;k]$, and thus $2b[n;k]$ bits of information on the kth subcarrier. One goal is thus formulated as the following constrained optimization problem:

$$\text{maximize } 2\sum_{k=0}^{K-1} b[n;k]$$

$$\text{subject to } \begin{array}{lll} c1 & \overline{BER}[n;k] = & \overline{BER}_0[k], \forall k \\ c2 & \sum_{k=0}^{K-1} P[n;k] = P_{total}\, and & P[n;k] \geq 0, \forall k \\ c3 & b[n;k] \varepsilon \{0,1,2,3,4,5,6,\ldots\}, & \end{array} \qquad (53)$$

where P_{total} is the total power available to the transmitter per block.

[0099] The constrained optimization in (10) calls for joint adaptation of the following parameter:

- power and bit loadings $\{P[n;k], b[n;k]\}_{k=0}^{K-1}$ across sub-carriers;
- basis-beams per subcarrier $\{u_1[n;k], u_2[n;k]\}_{k=0}^{K-1}$

- power splitting between the two basis-beams per subcarrier $\{\delta_1[n;k], \delta_2[n;k]\}_{k=0}^{K-1}$.

[0100] Compared with the constant-power transmissions over flat-fading MIMO channels, the problem here is more challenging, due to the needed power loading across OFDM subcarriers, which in turn depends on the 2D beamformer optimization per subcarrier. Intuitively speaking, our problem amounts to loading power and bits optimally across space and frequency, based on partial CSI.

Adaptive MIMO-OFDM With 2D Beamforming

[0101] For notational brevity, we drop the block index n, since our transmitter optimization is going to be performed on a per block basis. Our transmitter includes an inner stage (adaptive beamforming) and an outer stage (adaptive modulation). Instrumental to both stages is a threshold metric, $d_0^2[k]$, which determines *allowable combinations* of (P[k],b[k]), so that the prescribed $\overline{BER}_0[k]$ is guaranteed.

[0102] Next, the basis beams $u_1[k], u_2[k]$, and the corresponding percentages $\delta_1[k], \delta_2[k]$ of the power P[k] are determined for a fixed (but allowable) combination of (P[k], b[k]). Let Ts be the OFDM symbol duration with the cyclic prefix removed, and without loss of generality, let us set Ts = 1. With this normalization, the constellation chosen for the *k*th subcarrier has average energy $\varepsilon_s[k] = P[k]T_s = P[k]$, and contains $M[k] = 2^{b[k]}$ signaling points. If $d_{\min}^2[k]$ denotes the minimum square Euclidean distance for this constellation, we will find it convenient to work with the scaled distance metric $d^2[k] := d_{\min}^2[k]/4$, because for QAM constellations, it holds that,

$$d_{\min}^2[k] = 4d^2[k] = 4g(b[k])\varepsilon_s[k] = 4g(b[k])P[k], \tag{54}$$

where the constant *g*(*b*) depends on whether the chosen constellation is rectangular, or, square QAM:

$$g(b) := \begin{cases} \frac{6}{5\cdot 2^b - 4}, & b = 1,3,5,\ldots \\ \frac{6}{4\cdot 2^b - 4}, & b = 2,4,6,\ldots \end{cases} \tag{55}$$

[0103] Notice, that $d^2[k]$ summarizes the power and constellation (bit) loading information that the adaptive modulator passes on to the coder-beamformer. The later relies on $d^2[k]$ and

the partial CSI to adapt its design so as to meet constraint C1. To proceed with the adaptive beamformer design, we therefore need to analyze the BER performance of the scalar equivalent channel per subcarrier, with input $s_i[k]$ and output $z_i[k]$, as described by (48). For each (deterministic) realization of $h_{eqv}[k]$, the BER when detecting $s_i[k]$ in the presence of AWGN in (5), can be approximated as

$$BER[k] \approx 0.2\exp\left(-h_{eqv}^2[k]d^2[k]/N_0\right), \tag{56}$$

where the validity of the approximation has also been confirmed. Based on our partial CSI model, the transmitter perceives $h_{eqv}[k]$as a random variable, and evaluates the *average* BER performance on the kth subcarrier as:

$$\overline{BER}[k] \approx 0.2E\left[\exp\left(-h_{eqv}^2[k]d^2[k]/N_0\right)\right] \tag{57}$$

[0104] We will adapt our basis beams $\mathbf{u}_1[k]$, $\mathbf{u}_2[k]$ to minimize $\overline{BER}[k]$ for a given $d^2[k]$, based on partial CSI. To this end, we consider the eigen decomposition on the "nominal channel" per subcarrier (here the kth)

$$\begin{aligned}
&\overline{H}[k]\overline{H}^H[k] = \overline{U}_H[k]\Lambda_H[k]\overline{U}_H^H[k], with\\
&\overline{U}_H[k] := \left[\overline{u}_{H,1}[k], \ldots, \overline{u}_{H,N_t}[k]\right],\\
&\Lambda_H[k] := diag\left(\lambda_1[k], \ldots, \lambda_{N_t}[k]\right),
\end{aligned} \tag{58}$$

where $\overline{u}_H[k]$ is unitary, and $\Lambda_H[k]$ contains on its diagonal the eigenvalues in a non-increasing order: $\lambda_1[k] \geq \ldots \geq \lambda_{N_T}[k] \geq 0$. As proved, the optimal $\mathbf{u}_1[k]$ and $\mathbf{u}_2[k]$ minimizing the $\overline{BER}[k]$ are:

$$u_1[k] = \overline{u}_{H,1}[k], u_2[k] = \overline{u}_{H,2}[k], \tag{59}$$

Notice that the columns of $\overline{U}_{\mathrm{H}}[k]$ are also the eigenvectors of the channel correlation matrix $E\{\breve{H}[k]\breve{H}^{H}[k]\} = \overline{H}[k]\overline{H}^{H}[k] + N_r\sigma_{\varepsilon}^{2}[k]I_{N_t}$, that is perceived by the transmitter based on partial CSI. Hence, the basis beams $\mathbf{u}_1[k]$ and $\mathbf{u}_2[k]$ adapt to the two eigenvectors of the perceived channel correlation matrix, corresponding to the two largest eigenvalues.

[0105] Having obtained the optimal basis beams, to complete our beamformer design, we have to decide how to split the power $P[k]$ between these two basis beams.

[0106] With the optimal basis beams, the equivalent scalar channel is:

$$h_{eqv}^{2} = \delta_1\left\|\breve{H}^{H}[k]u_{H,1}[k]\right\|^{2} + \delta_2[k]\left\|\breve{H}^{H}[k]u_{H,2}[k]\right\|^{2}. \tag{60}$$

For $i = 1,2$, the vector $\breve{H}^{H}[k]u_{H,i}[k]$in (17) is Gaussian distributed with $CN\left(\overline{H}^{H}[k]u_{H,i}[k], \sigma_{\varepsilon}^{2}[k]I_{N_r}\right)$ Furthermore, we have that $\left\|\overline{H}^{H}[k]u_{H,i}[k]\right\|^{2} = \lambda_i[k]$. For an arbitrary vector $a \sim CN(\mu, \sum)$, the following identity holds true.

$$E\{\exp(-a^{H}a)\} = \exp\left(-\mu^{H}\left(I+\sum\right)^{-1}\mu\right)/\det\left(I+\sum\right). \tag{61}$$

Substituting (60) into (57), and applying (61), we obtain:

$$\overline{BER}[k] \approx 0.2\prod_{\mu=1}^{2}\left[\left(\frac{1}{1+\delta_{\mu}[k]d^{2}[k]\sigma_{\varepsilon}^{2}[k]/N_0}\right)^{Nr}\cdot\right]$$
$$\exp\left(-\frac{\lambda_{\mu}[k]\delta_{\mu}[k]d^{2}[k]/N_0}{1+\delta_{\mu}[k]d^{2}[k]\sigma_{\varepsilon}^{2}[k]/N_0}\right) \tag{62}$$

[0107] Eq. (62) shows that the power splitting percentages $\delta_1[k], \delta_2[k]$, depend on $\lambda_1[k], \lambda_2[k]$, and $d^2[k]$. Their optimum values can be found by minimizing (62) to obtain:

$$\delta_1[k] = \min\left(\overline{\delta}_1[k], 1\right), \quad \delta_2[k] = \max\left(\overline{\delta}_2[k], 0\right), \tag{63}$$

where, with $K_\mu[k] := \lambda_\mu[k]/(N_r\sigma_\varepsilon^2[k])$ and $m_\mu[k] := (1+K_\mu[k])^2/(1+2K_\mu[k])$, $\mu = 1,2$, we have

$$\bar{\delta}_\mu[k] = \frac{m_\mu[k]}{\sum_i m_i[k]} + \frac{m_u[k]}{d^2[k]\sigma_\varepsilon^2[k]/N_0} x \left(\frac{\sum_i \frac{m_i[k]}{1+K_i[k]}}{\sum_i m_i[k]} - \frac{1}{1+K_\mu[k]} \right), \mu = 1,2. \tag{64}$$

[0108] The solution guarantees that $0 \le \delta_2[k] \le \delta_1[k] \le 1$, and $\delta_1[k]+\delta_2[k]=1$. Based on the partial CSI $(\bar{\mathrm{H}}[k], \sigma_\varepsilon^2[k])$, eqns. (16) and (20) provide the 2D coder-beamformer design with the minimum $\overline{BER}[k]$, that is adapted to a given $d^2[k]$ output of the adaptive modulator. Because this minimum $\overline{BER}[k]$ depends on $d^2[k]$, the natural question at this point is: for which values of $d^2[k]$, call it $d_0^2[k]$, will the minimum $\overline{BER}[k]$ reach the target $\overline{BER}_0[k]$?

[0109] We next establish that $\overline{BER}[k]$ in (62), with $\{\delta_i\{k\}\}_{i=1}^2$ specified in (63), is a monotonically decreasing function of $d^2[k]$.

[0110] *Lemma*: Given partial CSI, the $\overline{BER}[k]$ in (62) is a monotonically decreasing function of $d^2[k]$. Hence, there exists a threshold $d_0^2[k]$for which $\overline{BER}[k] \le \overline{BER}_0[k]$ if and only if $d^2[k] \ge d_0^2[k]$. The threshold $d_0^2[k]$ is found by solving (19) with respect to $d^2[k]$, when $\overline{BER}[k] \le \overline{BER}_0[k]$.

[0111] *Proof*: A detailed proof requires the derivative of $\overline{BER}[k]$ with respect to $d^2[k]$, over two possible scenarios: $\delta_2[k]=0$, and $\delta_2[k]>0$, as indicated by (63). We have verified that this derivative is always less than zero for any given $d^2[k]$. However, we will skip the lengthy derivation, and provide an intuitive justification instead. Suppose that $\delta_1[k]$ and $\delta_2[k]$ are optimized as in (20) for a given $d^2[k]$. Now, let us increase $d^2[k]$ by an amount Δ_d. Even when $\delta_1[k]$ and $\delta_2[k]$ are fixed to previously optimized values (i.e, even if the 2D coder-beamformer is non-adaptive) the corresponding BER decreases, since signaling with

larger minimum distance always leads to better performance. With the minimum constellation distance $d^2[k]+\Delta_d$, optimizing $\delta_1[k]$ and $\delta_2[k]$ will further decrease the BER. Hence, increasing $d^2[k]$ decreases $\overline{BER}[k]$ monotonically.

[0112] This lemma implies that we can obtain the desirable $d^2[k]$. However, since no closed-form solution appears possible, we have to rely on a one-dimensional numerical search.

[0113] To avoid the numerical search, we next propose a simple, albeit *approximate*, solution for $d_0^2[k]$. Notice that eq. (62) is nothing but the average BER of an $2N_r$-branch diversity combining system, with N_r branches undergoing Rician fading with Rician factor $K_1[k]=\lambda_1[k]/(N_r\sigma_\epsilon^2[k])$; while the other N_r branches are experiencing Rician fading with Rician factor $K_2[k]=\lambda_2[k]/(N_r\sigma_\varepsilon^2[k])$. Approximating a Rician distribution by a Nakagami-*m* distribution, we can approximate the $\overline{BER}[k]$ by:

$$\overline{BER}'[k]\approx\frac{1}{5}\prod_{\mu=1}^{2}\left(1+\delta_\mu[k]\frac{(1+K_\mu[k])d^2[k]\sigma_\varepsilon^2[k]}{m_\mu[k]\cdot N_0}\right)^{-m_\mu[k]N_r}, \tag{65}$$

where m_μ is defined after eq. (63). It can be easily verified that $\overline{BER}'[k]$ is also monotonically decreasing as $d^2[k]$ increases. Setting $\overline{BER}'[k]=\overline{BER}_0[k]$, we can solve for $d_0^2[k]$ using the following two-step approach:

Step 1: Suppose that $d_0^2[k]$ can be found with $\delta_2[k]>0$. Substituting (64) into (65), we obtain:

$$d_0^2[k]=\left[\frac{A_0[k]\cdot\left(5\overline{BER}_0[k]\right)^{-1/(A_0[k]N_r)}}{\prod_{\mu=1}^{2}(1+K_\mu[k])^{m_\mu[k]/A_0[k]}}-B_0[k]\right]\cdot\frac{N_0}{\sigma_\varepsilon^2[k]}, \tag{66}$$

where

$$A_0[k]:=\sum_{i=1}^{2}m_i[k],\quad B_0[k]:=\sum_{i=1}^{2}\frac{m_i[k]}{1+K_i[k]}, \tag{67}$$

To verify the validity of the solution, let us substitute $d_0^2[k]$into (21). If $\bar{\delta}_2[k]>0$ is satisfied, then (66) yields the desired solution. Otherwise, we go to step 2.

Step 2: When Step 1 fails to find the desired $d_0^2[k]$ with $\delta_2[k]>0$, we set $\delta_2[k]=0$ Substituting $\delta_1[k]=1$ and $\delta_2[k]=0$, we have

$$d_0^2[k]=\frac{\left(5\overline{BER}_0[k]\right)^{-1/(m_1[k]N_r)}-1}{\left(1+K_1[k]\right)/m_1[k]}\cdot\frac{N_0}{\sigma_\epsilon^2[k]}, \tag{68}$$

[0114] This approximate solution of $d_0^2[k]$ avoids numerical search, thus reducing the transmitter complexity.

[0115] We next detail some important special cases.

[0116] *Special Case 1 – MIMO OFDM with one-dimensional (1D) beamforming based on partial CSI*: The 1D beamforming is subsumed by the 2D beamforming if one fixes a *priori* the power percentages to $\delta_1[k]=1$, and $\delta_2[k]=0$. In this case, $d_0^2[k]$ can be found in closed-form.

[0117] *Special Case 2 – SISO-OFDM based on partial CSI*: The single-antenna OFDM based on partial CSI can be obtained by setting $N_t = N_r = 1$. In this case, $\lambda_1[k]=\left|\bar{\mathrm{H}}[k]\right|^2$, where $\bar{\mathrm{H}}[k]$ is the "nominal channel" on the *k*th subcarrier. Hence, this yields $d_0^2[k]$in this case too, after setting $N_r = 1$, and $K_1 := \left\|\bar{\mathrm{H}}[k]\right\|^2/\sigma_\epsilon^2[k]$.

[0118] *Special Case 3 – MIMO-OFDM based on perfect CSI*: With $\sigma_\phi^2[k=0]$ the adaptive beamformer on each OFDM subcarrier reduces the ID beamformer with $\delta_2[k]=0$. This corresponds to the MIMO-OFDM system, when cochannel interference (CCI) is absent. In this special case, no Nakagami approximation is need, and the BER performance simplifies to

$$\overline{BER}[k]=0.2\exp\left(-d^2[k]\lambda_1[k]/N_0\right), \tag{69}$$

which leads to a simpler calculation of the threshold metrics as

$$d_0^2[k] = \left[ln\left(5\overline{BER}_0[k]\right)\right]N_0 / \lambda_1[k]. \quad (70)$$

[0119] *Special Case 4 – Wireline DMT systems*: The conventional wireline channel in DMT systems, can be incorporated in our partial CSI model by setting $N_t = 1$, $N_r = 1$, and $\sigma_\epsilon^2[k] = 0$. In this case, the threshold metric $d_0^2[k]$ is given by (70) with $\lambda_1[k] = |H[k]|^2$.

<u>*Adaptive Modulation based on Partial CSI*</u>

[0120] With $d_0^2[k]$ encapsulating the allowable ($P[k]$,$b[k]$) pairs per subcarrier, we are ready to pursue joint power and bit loading across OFDM subcarriers to maximize the data rate. It turns out that after suitable interpretations, many existing power and bit loading algorithms developed for DMT systems, can be applied to the adaptive MIMO-OFDM system based on partial CSI. We first show how the classical Hughes-Hartogs algorithm (HHA) can be utilized to obtain the optimal power and bit loadings.

1) Optimal Power and Bit Loading: As the loaded bits assume finite (non-negative integer) values, a globally optimal power and bit allocation exists. Given any allocation of bits on all subcarriers, we can construct it in a step by step bit loading manner, with each step adding a single bit on a certain subcarrier, and incurring a cost quantified by the additional power needed to maintain the target BER performance. This hints towards the idea behind the Hughes Hartogs algorithm (HHA): at each step, it tries to find which subcarrier supports one additional bit with the least required additional power. Notice that the HHA belongs to the class of greedy algorithms that have found many applications such as the minimum spanning tree, and Huffman encoding.

[0121] The minimum required power to maintain i bits in the kth sub carrier with threshold metric $d_0^2[k]$ is $d_0^2[k]/ g(i)$. Therefore, the power cost incurred when loading the ith bit to the kth subcarrier is

$$c(k,i) = \frac{d_0^2[k]}{g(i)} - \frac{d_0^2[k]}{g(i-1)}, \quad i \geq 1, \forall k. \quad (71)$$

[0122] For $i = 1$, we set $g(i-1) = \infty$, and thus $c(k,1) = d_0^2[k]/ g(1)$. In the following algorithm, we will use P_{rem} to record the remaining power after each bit loading step, $b_c[k]$ to

store the number of bits already loaded on the kth subcarrier, and $P_c[k]$ to denote the amount of power currently loaded on the kth subcarrier. Now we are ready to describe the greedy algorithm for joint power and bit loading of the adaptive MIMO-OFDM based on partial CSI.

The Greedy Algorithm:

1) Initialization: Set $P_{rem} = \mathrm{P}_{\mathrm{total}}$. For each subcarrier, set $b_c[k] = \mathrm{P}_c[k] = 0$ and compute $d_0^2[k]$.

2) Choose the subcarrier that requires the least power to load one additional bit; i.e., select

$$k_0 = \arg\min_{k} c(k, b_c[k]+1) \tag{72}$$

3) If the remaining power cannot accommodate it, i.e., if $P_{rem} < c(k_0, b_c[k_0]+1)$, then exit with $P[k] = P_c[k]$, and $b[k] = b_c[k]$. Otherwise, load one bit to subcarrier k_0, and update state variables as

$$\mathrm{P}_{rem} = \mathrm{P}_{rem} - c(k_0, b_c[k_0+1]), \tag{73}$$

$$\mathrm{P}_c[k_0] = \mathrm{P}_c[k_0] + c(k_0, b_c[k_0]+1), \tag{74}$$

$$b_c[k_0] = b_c[k_0] + 1. \tag{75}$$

4) Loop back to step 2.

[0123] The greedy algorithm yields a "1-bit optimal" solution, since it offers the optimal strategy at each step when only a single bit is considered. In general, the 1-bit optimal solution obtained by a greedy algorithm may not be overall optimal. However, for our problem at hand, we establish in Appendix I the following:

[0124] *Proposition 1*: The power and bit loading solution $\{\mathrm{P}[k], b[k]\}_{k=0}^{K-1}$ that the greed algorithm converges to, in a finite number of steps, is overall optimal.

[0125] Notice that the optimal bit loading solution may not be unique. This happens when two or more subcarriers have identical $d_0^2[k]$ under their respective (and possibly different) performance requirements. However, a unique solution can be always obtained, after establishing simple rules to break possible ties that may arise.

[0126] Allowing for both rectangular and square QAM constellations, the greedy algorithm loads one bit at a time. However, only square QAMs are used in may adaptive systems. If only square QAMs are selected during the adaptive modulation stage, we can then load two bits in each step of the greedy algorithm, and thereby halve the total number of iterations. It is natural to wonder whether restricting the class to square QAMs has a major impact on performance. Fortunately, as the following proposition establishes, limiting ourselves to square QAMs only incurs marginal loss:

[0127] *Proposition 2*: Relative to allowing for both rectangular and square QAMs incurs up to one bit loss (on the average) per transmitted space-time coded block, that contains two OFDM symbols.

[0128] Compared to the total number of bits conveyed by two OFDM symbols, the one bit loss is negligible when using only square QAM constellations. However, reducing the number of possible constellations by 50% simplifies the practical adaptive transmitter design. These considerations advocate only square QAM constellations for adaptive MIMO-OFDM modulation (this excludes also the popular BPSK choice).

[0129] The reason behind Proposition 2 is that square QAMs are more power efficient than rectangular QAMs. With K subcarriers at our disposal, it is always possible to avoid usage of less efficient rectangular QAMs, and save the remaining power for other subcarriers to use power-efficient square QAMs. Interestingly, this is different from the adaptive modulation over flat fading channels, where the transmit power is constant and considerable loss (on bit every two symbols on average) is involved, if only square QAM constellations are adopted.

2) *Practical Considerations*: The complexity of the optimal greedy algorithm is on the order of $O(N_{bits}K)$, where N_{bits} is the total number of bits loaded, and K is the number of subcarriers. And it is considerable when N_{bits} and K are large. Alternative low-complexity power and bit loading algorithms have been developed for DMT application. Notice that [4] and [19] study a dual problem: optimal allocation of power and bits to minimize the total transmission power with a target number of bits. Interestingly, the truncated water-filling solution can be modified and used in our transmitter design, while the fast algorithm can not, since it requires knowledge of the total number of bits to start with. In spite of low-complexity, the algorithm is suboptimal, and may result in a considerable rate loss due to the truncation operation.

[0130] The overall adaptation procedure for the adaptive MIMO-OFDM design based on partial CSI can be summarized as follows:

1) Basis beams per subcarrier $\{u_1[k], u_2[k]\}_{k=0}^{K-1}$ are adapted first using (59), to obtain an adaptive 2D coder beamformer for each subcarrier.
2) Power and bit loading $\{b[k], \mathrm{P}[k]\}_{k=0}^{K-1}$ is then jointly performed across all subcarriers, using the algorithm in [15] that offers optimality at complexity lower than the greedy algorithm.
3) Finally, power splitting between the two basis beams on each subcarrier $\{\delta_1[k], \delta_2[k]\}_{k=1}^{K}$ is decided using (63).

Examples

[0131] We set $K = 64$, $L = 5$, and assume that the channel taps are i.i.d. with covariance matrix $\sum_{\mu\nu} = \frac{1}{L=1} \mathrm{I}_{L=1}$ We allow for both rectangular and square QAM constellations in the adaptive modulations stage. Let the average transmit-SNR (signal to noise ration) across subcarriers is defined as: SNR = $P_{total}T_s/(KN_0)$. The transmission rate (the loaded number of bits) is counted every two OFDM symbols as: $\sum_{k=0}^{K-1} 2b[k]$.

Comparison between exact and approximate solution

[0132] Typical MIMO multipath channels were simulated with $N_t = 4$, $N_r = 2$, and $N_0 = 1$. For a certain channel realization, assuming 2D beamforming on each subcarrier, FIG. 13 plots the thresholds $d_0^2[k]$ obtained via numerical search, and from the closed-form solution based on eq. (65), with $p = 0.5, 0.8, 0.9$ and a target BER = 10^{-3}. FIG. 14 is the counterpart of FIG. 13, but with target BER = 10^{-4}. The non-negative eigenvalues $\lambda_1[k]$ and $\lambda_2[k]$ of the nominal channels are also plotted in dash-dotted lines for illustration purpose. Observe that the solutions of $d_0^2[k]$ obtained via these two different approaches are generally very close to each other. And the discrepancy decreases as the feedback quality p increases, or, as the target $\overline{BER}_0$ increases. Notice that the suboptimal closed-form solution in practice, some SNR margins may be needed to ensure the target BER performance. Nevertheless, the suboptimal closed-form solution for $d_0^2[k]$ will be used in the ensuing numerical results.

[0133] FIGS. 13 and 14 also reveal that on subchannels with large eigenvalues (indicating "good quality"), the resulting $d_0^2[k]$ is small; hence, large size constellations can be afforded on those subchannels.

Power and bit loading with the Greedy Algorithm

[0134] We set $N_t = 4$, $N_r = 2$, $\rho = 0.5$, SNR = 9 dB, and $\overline{BER}_0 = 10^{-4}$ For a certain channel realization, we plot the power and bit loading solutions obtained via the greedy algorithm in FIGS. 15 and 16, respectively. For illustration purpose, we also plot the threshold metrics $d_0^2[k]$. We observe that whenever there is a change in the bit loading solution in FIG. 16 from one subcarrier to the next, there will be an abrupt change in the corresponding power loading in FIG. 15. Furthermore, for those subcarriers with the same number of bits, the power loaded by the greedy algorithm is proportional to the threshold metric. Also, from the bit loading of the greedy algorithm in FIG. 16, we see that all subcarriers are loaded with an even number of bits (with the exception of one subcarrier at most),which is consistent with Proposition 2.

[0135] *Test case 3 – Adaptive MIMO OFDM based on partial CSI*: In addition to the adaptive MIMO-OFDM based on 1D and 2D coder-beamformers, we derive an adaptive transmitter that relies on higher-dimensional beamformers on each OFDM subcarrier; we term it any-D beamformer here. With $\overline{BER}_0 = 10^{-4}$, we compare non-adaptive transmission schemes (that use fixed constellations per OFDM subcarrier) and adaptive MIMO-OFDM schemes based on any-D, 2D, and 1D beamforming in FIG. 16 with $N_t = 2$, $N_r = 2$, in FIG. 18 with $N_t = 4$, $N_r = 2$, and in Fig. 8 with $N_t = 4$, $N_r = 4$. The Alamouti codes are used when $N_t = 2$, and the rate ¾ STBC code is used when $N_t = 4$. The transmission rates for adaptive MIMO-OFDM are averaged over 200 feedback realizations.

[0136] With $N_t = 2$ in FIG. 17, the any-D beamformer reduces to the 2D coder-beamformer, since there are at most two basis beams. With $N_t = 4$ in FIGS. 18 and 19, 23 observe that the adaptive transmitter based on 2D coder-beamformer achieves almost the same data rate as that based on any-D beamformer, for variable quality of the partial CSI (as p varies), and various size MIMO channels (as N_r varies). Thanks to its reduced complexity, 2D beamforming is thus preferred over any-D beamforming. On the other hand, the 1D

beamforming is considerably inferior to 2D beamforming when low quality CSI is present at the transmitter. But as CSI quality increases (e.g., $\rho \geq 0.9$), the transmitter based on 1D beamforming approaches the performance of that based on 2D beamforming.

[0137] With $N_t = 2$, $N_r = 2$ in FIG. 17, the adaptive MIMO-OFDM based on the 2D coder-beamformer always outperforms non-adaptive alternatives. With $N_t = 4$, $N_r = 2$ in FIG. 18, the non-adaptive transmitter at the low SNR range, with extremely low feedback quality ($\rho = 0$). However, as the SNR increases, or, the feedback quality improves, the adaptive 2D transmitter outperforms the non-adaptive transmitter considerably. As the number of receive antennas increase to $N_r = 4$ in FIG. 19, the adaptive 2D beamforming transmitter is uniformly better than the non-adaptive transmitter, regardless of the feedback quality.

Proofs

Based on (28) and (12) we have

$$c(k,i) = 2^{2(j-1)} d_0^2[k], \text{ for } i = 2j-1, 2j, \text{ and } j = 1,2,.... \qquad (76)$$

Table I lists the required power to load the ith bit on the kth subcarrier.

i	1	2	3	4	5	...
$d_0^2[k]/g(i)$	$d_0^2[k]$	$2d_0^2[k]$	$6d_0^2[k]$	$10d_0^2[k]$	$26d_0^2[k]$	...
$c(k,i)$	$d_0^2[k]$	$d_0^2[k]$	$4d_0^2[k]$	$4d_0^2[k]$	$16d_0^2[k]$	...

TABLE 1

From Table I and eq. (33), we infer that

$$c(k, i=1) \geq c(k,i), \quad \forall i,k . \qquad (77)$$

[0138] Although the greedy algorithm chooses always the 1-bit optimum, eq. (77) reveals that all future additional bits will cost no less power. This is the key to establishing the overall optimality because no matter what the optimal final solution is, the bits on each subcarrier can be constructed in a bit-by-bit fashion, with every increment being most power-efficient, as in the greedy algorithm. Hence, the greedy algorithm is overall optimal for our problem at hand. Lacking an inequality like (77), the optimality has been formally established.

[0139] An important observation from (76) is that $c(k, 2j-1) = c(k, 2j)$ holds true for any k and j. Suppose at some intermediate step of the greedy algorithm, the $(2j-1)$st bit on the kth subcarrier is the chosen bit to be loaded, which means that the associated cost $c(k, 2j-1)$ is

the minimum out of all possible choices. Notice that $c(k, 2j) = c(k, 2j-1)$ has exactly the same cost, and therefore, after loading the $(2j-1)$st bit on the kth subcarrier, the next bit chosen by the optimal greedy algorithm *must* be the $(2j)$th bit on the same subcarrier, unless power insufficiency is declared. So, the overall procedure effectively loads two bits at a time: as long as the power is adequate, the greedy algorithm will always load two bits in a row to each subcarrier. Let us denote the total number of bits as $R_{square} = 2\sum_{k=0}^{K-1} b_1[n;k]$, when using only square QAMs, and $R_{rect} = 2\sum_{k=0}^{K-1} b_2[n;k]$ when allowing also for rectangular QAMs. AT most on one subcarrier k', it holds that $b_2[n;k'] = b_1[n;k'] + 1$, which has probability ½; while for all other subcarriers, $b_2[n;k] = b_1[n;k] + 1$ Hence, R_{square} is less than R_{rect} by most one bit per space time coded OFDM block.

<u>Higher Than Two-D Beamforming</u>

[0140] For practical deployment of the adaptive transmitter, we have advocated the 2D coder-beamformer on each OFDM subcarrier. With $N_t > 2$ however, higher than 2D coder beamformers have been developed. They are formed by concatenating higher dimensional orthogonal space-time block coding designs, with properly loaded space time multiplexers. Collecting more diversity through multiple basis beams, the optimal N_t-dimensional beamformer outperforms the 2D coder-beamformer, from the minimum achievable $\overline{\text{BER}}$ point of view. Hence, with more than two basis beams, the threshold metric per subcarrier may improve, and the constellation size on each subcarrier may increase under the same performance constraint. However, the main disadvantage of N_t-dimensional beamforming is that the orthogonal STBC design loses rate when $N_t > 2$. The important issue in this context is how much one could lose in adaptive transmission rate by focusing only on the 2D coder-beamformer, instead of allowing all possible choices of beamforming that can use up to N_t basis beams.

[0141] In the following, we use the notation n_tD to denote beamforming with n_t "strongest" basis beams. With $n_t \leq 2$, two symbols are transmitted over two time slots as in (2). When $n_t = 3,4$, the beamformer can be constructed based on the rate ¾ orthogonal SBC, with three symbols transmitted over four time slots. When $5 \leq n_t \leq 8$, the beamformer can be constructed based on the rate ½ orthogonal STBC, with four symbols transmitted over eight

time slots. Let us consider, for simplicity, a maximum of eight directions even when $N_t > 8$, i.e., $n_{t,max} = \min(N_t, 8)$. If we take a super block with eight OFDM symbols as the adaptive modulation unit, then each super block allows for different n_tD beamformers on different subcarriers at each modulation adaptation step. Specifically, in one super block, one subcarrier could place four 2D coder-beamformers, or, two 4D beamformers, or one 8D beamformer, depending on partial CSI. With constellation size $M[k]$, the corresponding transmission rate for the n_tD beamformer is $8 f_{n_t} \log_2(M[k])$ per subcarrier per super block, where $f_{n_t} = 1$ for $n_t = 1,2$, $f_{n_t} = 3/4$ for $n_t = 3,4$, and $f_{n_t} = 1/2$ for $n_t = 5,6,7,8$. Furthermore, with power $P[k]$ on each subcarrier, the energy per information symbol is $d^2[k] = (1/f_{n_t}) g(b[k]) \mathrm{P}[k]$. This includes (11) as a special case with $f_1 = f_2 = 1$.

[0142] As with 2D beamforming, we wish to maximize the transmission rate of the MIMO-OFDM subject to the performance constraint on each subcarrier. We first determine the distance threshold $d_{0,n_t}^2[k]$ on each subcarrier for the ${}_{n_t}D$ beamformer, where $1 \le n_t \le n_{t,max}$. With the average BER expression for the n_tD beamformer, we find $d_{0,n_t}^2[k]$ through one dimensional numerical search. Hence, if the assigned constellation has $d^2[k] \ge d_{0,n_t}^2[k]$, adopting the n_tD beamformer will lead to the guaranteed BER performance, thanks to the monotonicity we established in our Lemma.

[0143] Having specified $\{d_{0,n_t}^2[k]\}_{k=0}^{K-1}$ for each $n_t \in [1,2,..,n_{t,max}]$, we can also modify our greedy algorithm, to obtain the optimal power and bit loading across subcarriers. First we define the effective number of bits $b_e := b f_{n_t}$ when 2^b-QAM is used together with n_tD beamforming. Second, we constrain the effective number of bits b_e to be integers, in order to facilitate the problem solving procedure. To achieve this, non-integer QAMs are assumed temporarily available for an n_t (we will later on quantize them to the closet square or rectangular QAMs). This entails a certain approximation error, but our objective here is to quantify the difference between 2D beamforming and any n_tD beamforming. The greedy algorithm can be applied as described, but with each step loading effectively one bit on certain subcarrier. Specifically, we need to replace $c(k,b_e+1)$ in the original greedy algorithm with $c(k,b_e+1)$, where

$$c(k, b_e + 1) = \min_{n_t}\left[\frac{f_{n_t} d_{0,n_t}^2[k]}{g((b_e + 1)/f_{n_t})}\right] - \min_{n_t}\left[\frac{f_{n_t} d_{0,n_t}^2[k]}{g(b_e / f_{n_t})}\right], \tag{78}$$

is the minimal power required to load one additional bit on top of b_e effective bits on the kth subcarrier, given that all possible n_tD beamformers can be arbitrarily chosen. Notice that the optimal beamforming, based on as many as $n_{t,max}$ basis beams, includes 2D beamforming as a special case with $n_{t,max} = 2$. Numerical results demonstrate that the 2D transmitter performs close to any higher dimensional one in most practical cases. However, the 2D transmitter reduces the complexity considerably, which is the reason why we favor the 2D coder-beamformer in practice.

Conclusion

[0144] The described MIMO-OFDM transmissions are capable of adapting to partial (statistical) channel state information (CSI). Adaptation takes place in three (out of four) levels at the transmitter: The power and (QAM) constellation size of the information symbols; the power splitting among space-time coded information symbol substreams; and the basis-beams of two- (or generally multi-) dimensional beamformers that are used (per time slot) to steer the transmission over the flat MIMO subchannels corresponding to each subcarrier.

[0145] For a fixed transmit-power, and a prescribed bit error rate performance per subcarrier, we maximize the transmission rate for the proposed transmitter structure over frequency-selective MIMO fading channels. The power and bits are judiciously allocated across space and subcarriers (frequency), based on partial CSI. Analogous to perfect-CSI-based DMT schemes, we established that loading in our partial-CSI-based MIMO OFDM design is controlled by a minimum distance parameter (which is analogous to the SNR-threshold used in DMT systems) that depends on the prescribed performance, the channel information, and its reliability, as those partially (statistically) perceived by the transmitter. This analogy we established offers two important implications: i) it unifies existing DMT metrics under the umbrella of partial CSI; and ii) it allows application of existing DMT loading algorithms from the wireline (perfect CSI) setup to the pragmatic wireless regime, where CSI is most often known only partially.

[0146] Regardless of the number of transmit antennas, the adaptive two-dimensional coder-beamformer should be preferred in practice, over higher-dimensional alternatives, since it enables desirable performance-rate-complexity tradeoffs.

[0147] Various embodiments of the invention have been described. The described techniques can be embodied in a variety of transmitters including base stations, cell phones, laptop computers, handheld computing devices, personal digital assistants (PDA's), and the like. The devices may include a digital signal processor (DSP), field programmable gate array (FPGA), application specific integrated circuit (ASIC) or similar hardware, firmware and/or software for implementing the techniques. In other words, constellation selectors and Eigen-beam-formers, as described herein, may be implemented in such hardware, software, firmware, or the like.

[0148] If implemented in software, a computer readable medium may store computer readable instructions, i.e., program code, that can be executed by a processor or DSP to carry out one of more of the techniques described above. For example, the computer readable medium may comprise random access memory (RAM), read-only memory (ROM), non-volatile random access memory (NVRAM), electrically erasable programmable read-only memory (EEPROM), flash memory, or the like. The computer readable medium may comprise computer readable instructions that when executed in a wireless communication device, cause the wireless communication device to carry out one or more of the techniques described herein. These and other embodiments are within the scope of the following claims.